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**SUBJECT: FURTHER MATHEMATICS CLASS: SSS3**

**SCHEME OF WORK**

**WEEK TOPIC**

**1. REVISION**

**2. PROBABILITY DISTRIBUTION**

Binomial Probability Distribution

Poisson Probability Distribution

**3. PROBABILITY DISTRIBUTION** (CONTINUATION)

Normal Distribution

Properties and Area

z – Scores Application.

**4. STATICS**

Definition of Concepts

Resultant of Two Forces

Components Resolution of Forces

**5. STATICS – CONTINUATION**

Definition of Equilibrium and Condition of Equilibrium of Rigid Body

Application of the Condition to Solve Problems

Lami’sTheorem and Application

**6. REVIEW OF WEEK 1 – 5**

**7. STATICS – CONTINUATION**.

Definition of Moment of a Force

Principles of Moments

Application of the Principle in Solving Problems

**8**. **FRICTION.**

Basic Concept

Coefficient of Friction

Forces Actingon a body.

**REFERENCE TEXTBOOK**

FURTHER MATHEMATICS PROJECT 3

**WEEK TWO**

**PROBABILITY DISTRIBUTION**

* BINOMIAL PROBABILITY DISTRIBUTION
* POISSON PROBABILITY DISTRIBUTON

Probability distribution deals with theoretical probability model based on the randomness of certain natural occurrences. The binomial and Poisson distribution are discrete distribution

**BINOMIAL DISTRIBUTION**

This arises from a repeated random experiment which has two possible outcomes.

The two possible outcomes of the random experiment are usually called success and failure.

Prob( success) = P, Prob(failure) = q

Since the two events are complementary, hence p+ q = 1 or p = 1-q, q = 1 – p

The probability of success or failure of an event is the same for each trials and does not influence the probability of success or failure of another trial of the same event.

:. Binomial distribution of n trails and r required outcome(s) is defined as :

pr(x = r) = nCrPrqn-r

whennCr = n!

(n-r)! r!.

The binomial distribution is suitable when the number of trials is not too large.

**Example:**

1. Find the probability that when two fair coins are tossed 5 times a head and a tail appear three times.

Solution:

Two fair coins = (HT, TH, TT, HH) = 4

Prob (a head and a tail) = 2/4 = ½

i.e p = ½ , q = ½ (p + q = 1)

n = 5, r = 3.

:. P(x = r) =nCrprqn-r

p ( x = 3) = 5C3 ( ½ ) 3 ( ½ ) 5-3

p (x = 3) = 10 x 1/8 x ¼ = 10/32 = 5/16

p (x = 3) = 0.3125.

2. It is known that 2 out of every 5 cigarettes smokers is a village have cancer of the lungs. Find the probability that out of a random sample of 8 smokers from the village, 5 will have cancer of the lungs.

**Solution**

Prob( a smoker has cancer) = 2/5i.e p = 2/5

Prob( a smoker doesn’t have cancer) = 1 – 2/5 = 3/5

:. q = 3/5

n = 8, r = 5

Prob(x = -5) =8C5 (2/5)5 (3/5)3

= 56 x 32 x 27 = 48384

3125 125 390625

Prob (x = 5) = 0.124.

**EVALUATION**

Find the probability that when a fair six-faced die is tossed six times, a prime number appears exactly four times.

**POISSON DISTRIBUTION**: The Poisson distribution is more suitable when the number of trials is very large and probability of successes is small. It is defined as:

Pr(x) = λx e- λ , x = 0, 1, 2, 3,

x!

Where λ = np e = 2.718

P = probability of success, n = number of trials.

**Example:**

If 8% of articles in a large consignment are defective, what is the chance that 30 articles selected at random will contain fewer than 3 defective articles?

**Solution**

P = 8/100 = 0.08, n = 30

:. λ = np = 0.08 x 30 = 2.4.

Prob( fewer than 3) i.eprob( (0) + prob (1) + prob (2)

Prob (x = 0) =2.4o x e-2.4 = 1 x e-2.4

0!

Prob(x = 1) = 2.4o x e-2.4 = 2.4 x e-2.4

1!

Prob(x = 2) =2.4o x e-2.4 = 2.88 x e -2.4

2!

Prob(x <3) = e -2.4 + 2.4 x e-2.4 + 2.88 x e-2.4

= e-2.4 (1 + 2.4 + 2.88)

= e-2.4 x 6.28.

**EVALUATION**

The probability that a person gets a reaction from a new drug on the market is 0.001. If 200 people are treated with this drug. Find approximately, the probability that:

1. exactly three persons will get a reaction
2. more than two person will get a reaction

Properties of Binomial and Poisson Distribution.

**Binomial**

It assigns probability to non-occurrence of events i.eProb( x = 0)

Mean µ = np

Standard deviation, r = √npq

Variance ð2 = npq

**Poisson**

It assigns probability to non-occurrence of events i.eProb(x = 0)

Mean µ = λ = np

Standard deviation, ð = √λ = √np

Variance, ð2 = λ = np

Example:

1. In the probability of tossing a fair coin three times, a head shows up twice. Find the mean and standard deviation.

**Solution**

n = 3 Prob(a head) = ½ , i.e p = ½ , q = ½

I. Mean µ = np = 3 x ½ = 3/2

II. Standard deviation :r = √npq = 3 x ½ x ½ = ¾

Example

2. 0.2% of the cooks produced by a machine were found to be defective. If there are 1000 corks, find the mean and standard deviation?

**Solution**

P = 0.2% = 0.002.

N = 1000

I. mean µ = λ = np = 1000 x 0.002 = 2

II. r = √np = √2

**EVALUATION**

In an examination, 60% of the candidates pass. If 10 candidates were sampled. Find the mean, standard deviation and variance of the candidates.

**GENERAL EVALUATION**

1. The probability that a person gets a reaction from a new drug in the market is 0.001. If 2000 people were treated with this drug, find the mean and standard deviation.
2. 1. In an examination, 60% of the candidates passed. Use the binomial distribution to calculate the probabilities that a random sample of 10 candidates contain exactly 2 failures.

**READING ASIGNMENT**

Read probability distribution, further math. Project 3 from page 198-201.

**WEEKEND ASSIGNMENT**

1. What is the variance of a binomial distribution?

(a) np (b) √npq ( c ) npq (d) p2

2. The mean (µ) of a poisson distribution is the same as

(a) Standard deviation (b) variance (c) mean (d) mean deviation

3. If number of trials is 100 and probability of success is 0.0001, what is the variance of this distribution?

(a) 0.00999 (b) 0.1 (c ) 0.01 (d) 0.001

4. If the birth of a male child and that of a female child are equiprobable. Find the probability that in a family of five children exactly 3 will be male. (a) 16/5 (b) 5/16 (c) 5/32 (d) 5/21

5. If an unbiased die is thrown repeatedly, what are the chances that the first, six to be thrown will be the third throw? (a) 25/216 (b) 1/6 (c) 25/36 (d)25/31

**THEORY**

1. 20% of the total production of transistors produced by a machine are below standard. If a random sample of 6 transistors produced by the machine is taken, what is the probability of getting

(i) exactly 2 (ii) exactly 1 (iii) at least 2 (iv) at most 2 standard transistors?

2. A fair die is thrown five times. Calculate correct to 3 decimal places, the probability of obtaining

(a) at most two sixes (b) exactly three sixes

**WEEK THREE**

**PROBABILITY DISTRIBUTION (CONTINUATION)**

* Normal distribution
* properties and area
* z – scores application.

Normal Distribution: This is a continuous distribution and it takes the form

P(x) = 1 e-½ (x – u)2

ð√2xð

where∂ is the standard deviation. U is the mean and e = 2.718.

The graphical representation of a normal distribution is a bell-shaped curve.

P(x)

u

**PROPERTIES OF THE NORMAL DISTRIBUTION**

1. It depends on the mean (u) and standard deviation
2. The shape is bell-shaped
3. The function is continuous, hence the range is from –ά to + ά
4. The curve is symmetrical about the vertical line through the mean.

A normal distribution function is a probability function, hence the total area under the curve is 1 .

The normal distribution has a complicated equation, but it can be shown in shaded area under the shape

1. Values within 1 sd of the mean

Pr (μ – r <x ≤µ + r) = 0.68.

x

µ-ðµ µ+ð

2. Values within 2 sd of the mean

Pr(μ-2r <x < μ+ 2R) = 0.955

µ– 2r µ µ+2r

3. Values within 3sd of the mean.

Pr(μ-3r <x < μ + 3R) = 0.997

u– 2r µ µ+2r

Example: A random variable X is normally distributed with mean 65 and standard deviation 5, find:

1. Pr (60 ≤ x≤ 70) II. Pr (55 ≤ x ≤ 75) III. Pr(50 ≤ x ≤80 )

Solution:

µ = 65, r = 5

1. Pr (60 ≤ x ≤ 70) = Pr (µ - r ≤ x ≤ µ + r)

= Pr (65-5≤x ≤ 65+5)

= Pr (60 ≤ x ≤ 70) = 0.68

II. Pr (55 ≤ x ≤ 75) = Pr (µ -2r ≤ x ≤ µ + 2r)

= 0.95

III. Pr (50 ≤ x ≤ 80) = Pr (µ -35 ≤ x ≤ µ +3 r)

= 0.997

**EVALUATION**

1. A random variable x is normally distributed with mean 45 and standard deviation 12.

Find: I Pr (9 < x <81) II. Pr(33 < x < 57)

**AREA UNDER NORMAL CURVE**

The area under a normal curve can be defined by checking the probability value in the Normal distribution probabilities table.

Example:

1. Find the area between Z = 0 and Z = 2.13

Solution:

Pr( 0 < z < 2.13) check the value against 2.13.

Pr( 0 < z < 2.13) = 0.4834

0 2.13

2. Find the area z = -1.3 and z = 1.2

**Solution**

Pr (-1.3 < z<1.2)

= Pr (0 < z<1.2) +Pr (0 < z<1.3)

= 0. 3849 + 0.4032.

= 0.7881.

3. Find the area between Z = 0.36 and Z = 1.89

**Solution**

Pr (0.36 < z<1.89)

=Pr (0 < z<1.2) - Pr (0 < z<0.36)

= 0.4706 – 0.1406

= 0.33.

**EVALUATION**

Using the standard deviation normal distribution table. Find the area under

1. Pr (-1.5 < z< 2.0) 2.Pr(z <1.5) 3.Pr (z >2.6)

**Z Scores**

The area under a normal distribution curve between two values depends on the number of standard deviations from the mean. Therefore, the standardize normal curve is obtained from the normal curve by the substitution.

Z = X - µ

σ

:. Z is called the standardized score or Z score at mean zero (o) and standard deviation 1.

Example: A random variable whose distribution is normal has mean 25 and standard deviation 5. Find

I. Pr (22 < z< 27) II.Pr (x <20) III.Pr (x > 26.5)

**Solution**

µ= 25, = 5

I. Pr(22 < x < 27) = Pr ( x1-µ<z <x2 - µ)

σσ =Pr(z1< z< z2)

Z1= 22 – 25 = -3= - 0.6

5 5

Z2 = 27-25 = 2/5 = 0.4

5

Pr (-1.3 < z<1.2) = Pr (-0.6 < z<0.4)

= Pr (z <0.4) + Pr (z<0.6)

= 0.1554 + 0.2258 = 0.3812

II. Pr(x <20) = Pr (z< 20-25)

5

=Pr (z<-1) = Pr (-1 < z<0)

=Pr (z< 0) – Pr (z<1)

= 0.5 – 0.3413.

:. Pr(x <20) = 0.1587

III. Pr(x > 26.5) = = Pr (z>26.5 - 25)

5

= Pr (z > 0.3) = Pr (0.3 < z < ∞)

=Pr (z < ∞) – Pr (z < 0.3)

= 0.5 - 0.1179

= 0.3821.

**EVALUATION**

The weights of packets of sugar produced by a machine have a mean of 1kg and a standard deviation of 0.1kg. What is the probability that in a random sample of 50 packets the combined weight will exceed 52kg?

**GENERAL EVALUATION**

1. The inner diameters of bolts produced in a factory are normally distributed with mean 5cm and standard deviation 0.02cm. Find (a) the percentage of the number of bolts with inner diameters less than 5.015cm; (b) the probability that a bolt will have an inner diameter between 4.995cm and 5.015cm.
2. Use the standard normal distribution table to find (i) Pr (Z > 2.6) (ii) Pr (- 1.5 < Z < 1.7)

**READING ASIGNMENT**

Read Z scores and Normal distribution. Further Mathematics Project III Page 2 202-210.

**WEEKEND ASSIGNMENT**

1. Find the area between z = 0.36 and z= 1.89 (a) 0.33 (b) 0.6112 (c) 1.00

Use the information below to answer questions 2-4.

A distribution with mean 85 and standard deviation 10 is normally distributed. If x is a random variable of the distribution, find

2. Pr (80 < x <8.9) (a) 0.9332 (b) 0.5 (c) 0.3469

3. Pr(x >83) (a) 0.1587 (b) 0.789 (c) 0.4207

4. Pr(x > 87) (a) 0.0047 (b) 0.35 (c) 0.4207

5. Find, with the usual notations, P (z <1.810) from the table of normal distribution.

(a) 0.311 (b) 0.0288 (c) 0.9649

**THEORY**

1. The scores of some 500 candidates in an examination were found to be approximately normally distributed with mean 40 and standard deviation 5. Find the number of candidates who scored at least 48.
2. The lengths of nails produced in a factory are approximately normally distributed with mean 2cm and a standard deviation 0.01cm. Find the proportion of nails that will be shorter than 1.98cm.

**WEEK FOUR**

**STATICS**

* Definition of Concepts
* Resultant of two forces
* Components resolution of forces

**DEFINITION OF CONCEPT**

Statics is the study of bodies which remain at rest under the action of given forces.

**Mass:** This is the quantity of matter contain in a body. Mass of a particular body does not change and the standard unit is kilogram.

**Force:** Force is that action which tends to change the state of rest or uniform motion of a body in a straight line. It’s a vector quantity sine it has magnitude and direction. The unit of force is Newton.

:. F = Ma

Where M = Mass, a = acceleration.

Composition of Forces: Two or more concurrent forces can be combined to obtain a single force. Therefore, resultant force is the force produced or obtained when two or more concurrent forces are combined.

A force can be resolve by

I Graphical Method II. Analytical Method.

**Analytical Method**: The parallelogram law of composition of two forces is used to find the resultant force of two or more forces. Hence, parallelogram law states that if two forces acting at a point are represented in magnitude and direction by two adjacent sides of a parallelogram, then the resultants of the two forces, is represented in magnitude and direction by the diagonal of the parallelogram, drawn from the point of action of the two forces.

A C

R

θ

B D

R is the resultant force and can be obtained using cosine rule:

R2 = P2 + Q2 – 2PQcos(180 –θ)

The angle of inclination is ά and can be obtained using sine rule or tan.

:. Tan ά = CD

OD

Tan ά = P Sin θ

Q + P Cos θ

**Example 1**: The angle between two forces of magnitude 8N and 5N is 1200. Find in N, the magnitude of their resultant.

Solution

P R

5N R

Q 8N S

Let R be the resultant vectorial force.

R2 = P2 + Q2 – 2PQ Cos R.

= 52 + 82 – 2 (5 x 8) Cos 60o

= 25 + 64 – 80 x ½

= 89 – 40

R2 = 49

:. R = √49 R = 7N

2. Calculate, correct to one decimal place, the angle between two forces 20N and 30N if their resultant is 40N.

R2 = P2 + Q2 – 2PQ Cos (180 – θ)

402 = 202 + 302 – 2 (20 x 30) Cos (180 – θ)

1600 = 400 + 900 – 1200 cos(180 – θ)

1600-1300 = 1200Cos (180 –θ)

300 = -1200 Cos (180 – θ)

1. = Cos ( 180 –θ )

-1200

-1/4 = Cos (180 – R)

- 180 –θ = Cos -1(-1/4)

180 – θ = 104. 5

180 – 104.5 = θ

θ = 75.5

:. The angle between them is 75.5o

**EVALUATION**

The angle between two forces of magnitude 8N and 11N is 35o. Find the magnitude and inclination to the 11N force of resultant force.

**RESOLUTION OF FORCES**

A given force can be resolved into two parts and each part is called the resolute. A given force can be resolve in two directions which are perpendicular to each other.

The component along the y axis is called the vertical component and the component along the x- axis is the horizontal component.

y

pyPy

θ

PxPx

Let Px the horizontal component

Let py the vertical component.

If p is inclined to the upward vertical

Px Sin θ = pxCos θ = py

PP

Px = P sin θ. Py = P cos Ө.

Py P1

P4

py

px

-px px

P3 Py P2

Force Horizontal Component Vertical Component

P1 P1 Sin θ P1 Cos θ

P2 P2cos θ -P2 Sinθ

P3 -P3 cos θ - P3 Sin θ

P4 - P4 cos θ P4 sin θ

Resultant of several concurrent forces :the resultant is obtained as :

R = √(∑Px)2 + (∑Py)2

Where ∑Px = Sum of horizontal components ∑Py = Sum of vertical components

Tan θ =Py

Px

Θ = tan -1 ∑Py

∑Px Angle of inclination to the resultant.

**Example**:

1.The horizontal component of a force p which makes an angle of 50o with the horizontal is 30N. Find the force P.

**Solution**

30 = P x 0.6 + 28

P = 30/0.6428

P = 46.67N

2.Three forces (2N, 0600), ( 4.5N, 1800) and (5N, 3000) act on a body of mass 2kg which is initially at rest. Find I the resultant force on the body II the acceleration with which the body begins to move.

**Solution**

Let P1 = (2N.0600). P2 = (4.5N, 1800) P3 = (5N, 3000)

5N 2N

4.5N

Horizontal Component Vertical Components

P1 = 2 Sin 600 = 1.7321 P1 = 2 Cos 600 = 1

P2 = 4.5 Cos 900 = O P2 = -4.5 Sin 900 = -4.5

P3 = 5Cos 300 = -4.33 P3 = 5 Sin 300 = 2.5

∑px = 1.732 + 0 – 4.33 ∑py = 1 – 4.5 + 2.5

= - 2.598 ∑py = -1

(∑Px)2 = 6.7496 (∑py)2 = 1

:. R =√ (∑Px )2+ (∑Py)2= √6.7496 + 1 = √7.7496

R = 2.78N

1. Acceleration of the body ; F = ma

M = 2kg F = Resultant force = 2.78

F = ma

* 1. = 2 x a

a = 2.78/2  a = 1.39ms-2

**EVALUATION**

The vertical component of a force F which makes an angle of 350 with the horizontal is 45N. Find the force F.

**GENERAL EVALUATION**

1. The forces of magnitude 35N and 45N act on a particle in the directions 1800 and 3150 respectively. Find the resultant of these forces giving:

a. the magnitude correct to the nearest whole number b. the direction correct to the nearest degree.

1. A vertical force of 6N and a horizontal force of 8N act on a body. Find the magnitude and the inclination of the resultant force to the horizontal.

**READING ASIGNMENT**

Read Composition and Resolution of Coplanar forces on pages 154 to 165 of further mathematics project III.

**WEEKEND ASSIGNMENT**

1. Two forces each of magnitude PN are inclined to each other at an angle of 1200. Find the magnitude of their resultant.

(a) P√3 N (b) P2N ( c) PN

2. Find the angle between the two forces 5N and 6N if their resultant is 8N.

(a) 600 (b) 1200 (c) 1800

3. A force P of magnitude 60N makes an angle of 400 with the horizontal. Use the information to answer questions 3 and 4

3. Find the horizontal component of P

(a) 20N (b) 45.96N (c ) 38.57N

4. Find the vertical component of P

(a) 45.96N (b) 38.57 (d) 20N.

5. Find the resultant of forces 8N and 10N inclined at an angle 1200 to each other.

(a) 2√61N (b) 61√2N C 39N

**THEORY**

1. Forces F1 = (10N, 0900), F2 = (20N, 2100) and F3 = (4N, 3300) act on a body at rest on a smooth table. Find, correct to one decimal place the magnitude of the resultant force.
2. Find the magnitude and direction of the resultant of the forces shown in the diagram below:

6N 8N

10N

**WEEK FIVE**

**STATICS – CONTINUATION**

Definition of equilibrium

Condition of equilibrium of rigid body

Application of the condition to solve problems

Lami’s theorem and application

**Definitions**

Equilibrium is when a body remains at rest under the action of given forces.

**Translational Equilibrium**: The state of equilibrium of bodies which remain at rest under the action of forces have tendency to cause translation.

**Condition of Equilibrium**

When a block is placed on a table as shown below and force F1 and F2 are applied to the block.

The block remains in translational equilibrium if the magnitude of F1 and F2 are equal.

Since F1 and F2 are acting in opposite direction, but have equal magnitudes.

Then, F1 = -F2

F1 + F2 = 0

Also, the upward force N balances the downward force mg on the block.

:. N = -mg, N + mg = 0

Hence, the sum of the vertical components and horizontal components of forces acting on a body in translational equilibrium is equal to zero.

**Example**

1. A particle of mass 5kg is supported by two light inelastic strings inclined at angles 300 and 450 respectively to the horizontal. If the system is in equilibrium, calculate the tension in each string.

Solution:

30o

45o

5kg

Let the two inelastic strings to be DP and OQ

Resolve each string vertically and horizontally:

Horizontal : Efx = T1Cos 300 + T2cos 450 + 0

Vertical : Efy = T1 Sin 300 + T2 sin 450 – 50

:. Efx = 0.7061T2 – 0.866T1 = 0 equation I

Efy = 0.7071T2 + 0.5T1 = 50 equation II

Solving equation I and II simultaneously:

* 1.366T1 = -50

T1 = -50/-1.366 T1 = 36.6N

Substitute T1 in equation I or II using equation 1, 0.7071T1 – 0.866T1 = 0.

0.7071T2 = 0.866 x 36.6

T2 = 31.6956

0.7071

T2 = 44.82N

:. The tensions in the strings are T1 = 36.6N, T2 = 44.82N

2. A particle of mass 5kg is suspended by a light in extensive string which makes an angle of 300 with the downward vertical and horizontal forced F. If the system is in equilibrium, calculate

1. The tension in the string (ii) The magnitude of the force F (take g = 10ms-2)

**Solution**

30o

F

I using Sin θ = AB

OA

Sin 600 = 5 x 10

T

T = 50/sin 600, T = 57.74N. The tension in the string is 57.74 N

II. Magnitude of F1 Tan 300 = OB

AB

50 x Tan 300 = F :. F = 28.87N

**EVALUATION**

1.A street lamp of mass 10kg is suspended at a position ) by two wires OP and OQ across a rood such that each wire is inclined at an angle of 800 to the upward vertical, If the system is in equilibrium, calculate the tension in one of the two wires. (Take g = 10ms-2)

**TRIANGLE OF FORCES**

If three coplanar forces act on a body in such a way that the system is in equilibrium, then the forces can be represented in magnitude and direction by the sides of a triangle taken in order

The triangle representing the three coplanar forces is called a triangle of forces.

**Example**:

1.A body of mass 6.5kg is supported by two strings, One of the stings is inclined at an angle of 300 and the other 400 to the horizontal . Find the tension in each strings, if the system is in equilibrium (take g = 10ms-2)

**Solution**

30o

45o

6.5kg

Using Sine rule;

T1 = 65

Sin 50o Sin 70o

T1 = 65 x sin 50o

Sin 50o Sin 70

T1 = 65 x 0.766

0.9397

T1 = 52.98N

Similarly:

T2 = W

Sin 600 Sin 700

T2, = 65 x sin 600 = 65 x 0.8660

Sin 700 0.9397

T2 = 59.9N

**EVALUATION**

A body of mass s10kg is suspended by means of two light inextensible strings. AP and BP which are inclined at angles 600 and 300 respectively to the downward vertical. If T1 and T2 are the magnitude of the tension AP and BP respectively, calculate the values of T1 and T2.

**LAMI’S THEOREM**

This theorem states that if three forces acting at a point are in equilibrium, then each force is proportional to the sine of the angle between the liens of action of the other two forces.

Consider the forces F1, F2 and F3 below

By Lami’s theorem: F1 α Sin ***B*F2 α** sin *γ*F3 α sin θ

Since the forces are proportional to the sine of the angle: then F1 = F2 = F3

Sin *B*sinγ sin θ

Example:

1. A body of weight 91N is suspended by two inelastic string 5m and 12 m long attached to two points on the same horizontal level, whose distance apart is 13m. Using Lami’s theorem or otherwise, find the tension along the strings.

13m

5m

91N

12m

**Solution**

s

Le the tension in the strings be T1nd T2 and x, B be the angles made with the horizontal.

AB2 = OA2 + OB2 = 52 +122

= 25 + 144 =169

but AB = 13

AB2 = 132 = 169

:. OA2 + OB2 = AB2

hence AOB is a right angle triangle

:. Sin B = 5/13, sin x = 12/13

B = sin -1 0.3845 Ө = sin-1 12/13

B = 22.60 = 0.9231 = 67.40

Using Lami’s theorem:

T1 = W

Sin 157.4 sin 900

T1 = 91 x sin 157.40 = 91 x 0.3843

Sin 900 1

T1 = 34.97N

T2 = W

Sin 112.6 sin 900 T2 = 91 x sin 112.6

Sin 900

T2 = 91 x 0.9232

1

T2 = 84.01N.

**EVALUATION**

A particle of mass 10kg is connected by two strings of length 3m and 4m to two points on the same horizontal level and 5m apart, find the tension in the strings.

**GENERAL EVALUATION**

1. A particle of mass 98kg is suspended by two light inelastic strings of length 9m and 12m from two fixed point P and which are 15m apart. Calculate (i) the angles made by the strings with the upward vertical (ii) the tension in the strings.
2. The ends P and Q of an inextensible string 17m long are attached to two fixed points 13m apart on the same horizontal level. A body of mass 20kg is suspended from a point C on the string 5m from P. Calculate (a) the angle which each part of the string makes with the horizontal (b) the tension in each part of the string.

**READING ASIGNMENT**

Read Equilibrium “pages” 170-177 of Further Mathematics Project III.

**WEEKEND ASSIGNMENT**

Two forces (8N, 0300) and (10N, 1200) act on a body; find the magnitude of the force that would be applied to keep the system in equilibrium.

**WEEK SIX**

**REVIEW OF FIRST HALF LESSONS**

**WEEK TWO**

In a community, 10% of the people tested positive to the HIV virus. If 6 persons from the community

are selected at random, one after the other with replacement, calculate correct to four decimal places,

the probability that (i) exactly 5 (ii) none (iii) at most 2, tested positive to the virus.

**WEEK THREE**

The mean score of 200 students in an examination is 40 and the standard deviation is 8. if the scores

are assumed to be normally distributed, find the :

1. Proportion of students obtaining more than 46
2. Number of students scoring between 32 and 48.

**WEEK FOUR**

Coplanar forces 4N, 8N, 6N, 4N and 5N act at a point as shown in the diagram . If the 6Nforce act in the direction 0900 calculate the: (a) Magnitude of the resultant force (b) Direction of the resultant force

**WEEK FIVE**

A uniform plank PQ of length 8m and mass 10kg is supported horizontally at the end P and at point R. 3 metres from Q. A boy of mass 20kg walks along the plank starting from P. If the plank is in equilibrium, calculate the

a. reaction at P and R when he has walked 1.5 metres

b. distance he had walked when the two reactions are equal. (take g = 10ms-2)

**WEEK FIVE**

**STATICS – CONTINUATION**

Definition of Moment of a Force

Principles of Moments

Application of the Principle in Solving Problems

**Definition**

**Moments of a Force**: The moment of a force about a reference point is defined as the product of the force and the force arm. Moment of a force is a vector quantity and its units is Nm.

The direction of movement or sense of movement about the point is duly considered.

The object can move in clockwise moment and ant-clockwise moments about the given point.

Suppose we have an object acted upon by two forces, F1 and F2 in the opposite direction, then

M1 = F1 x di M2= F2 x d2

Where M1 = Magnitude of F1 M2 = Magnitude of F2

d1 = Force arm of d1 d2 = Force arm of d2

PRINCIPLE OF MOMENTS

1. When a system of coplanar forces is in equilibrium, then the sum of the clockwise moment is equal to the sum of the anti-clockwise moments about the same point in the plane.

2. When the system is not in equilibrium, then the resultant of two coplanar forces F1 and F2 denote by R is represented by the relationship below:

M1 + M2 + MR

Where M1 = Moment of F1 M2 = Moments of F2 MR = Moments of T.

**Centre of Gravity**: The centre of gravity of a uniform plank or rod is the midpoint of the plant or rod

**Examples**

1. A uniform rod PQ is 15m long and has mass 20kg. The rod rests on two supports at P and Q. An object of mass 5kg is suspended at a point R on the rod 5m from the end P. Calculate the reaction of the supports P and Q (take g = 10ms-2)

Solution

Kpkq

7.5m

5m

P Q

5kg 20kg

Let the reaction at P be Kp and at Q beKq

NB: The weight of the rod acts downward through the midpoint of the rod.

Moments about the point P, MR = M1 + M2

But M1 = F1 x d1 and F = Mg.

Kq x 15 = (5 x 10 ) x 5 + ( 20 x 10 ) x 7.5

15kq = 50 x 5 + 200 x 7.5

Kq = 250 + 1500

15.

Kq = 116. 7N.

Moment about the point Q,

Kp x 15 = ( 5 x 10 ) x 10 + (20 x 10 ) x 7.5.

15Kp = 500 + 1500

Kp = 2000

15.

Kp = 133.3N

1. A uniform beam AB of length 6m and mass 20kg rests on support P and Q placed 1m from each end of the beam. Masses of 10kg and 8kg are placed at A and B respectively. Calculate the reactions at P and Q (g = 9.8ms-2)

**Solution**

8kg

10kg

20kg

Let reaction at point P be Rp and at point Q be Rq.

:. Moment about the point Q

Rp x 4 = ( 10 x 9.8) x 5 + (20 x 9.8 ) x 2 – ( 8 x 9.8 x 1 )

4Rp = 490 + 392 - 78.4

Rp = 803.6

4. Rp = 200.9N

Moment about the point P:

Rq x 4 = ( 8 x 9.8 ) x 5 + (20 x 9.8 ) x 2 - ( 10 x 9.8) x 1

4Rq = 392 + 392 – 98

Rq = 689

4

Rq = 171.5N

**EVALUATION**

A uniform rod PQ, is 20 m long and weighs 80N, has weights 20 N and 50N suspended at P and Q respectively. Find the distance from P where the rod must be supported so that it will rest horizontally.

**GENERAL EVALUATION**

A uniform rod PQ of length 10 m and mass 2kg rest on two supports at x and y. If PX = 2m and QY = 1m, find the reaction of X. (Take g = 10ms-2)

**WEEKEND ASSIGNMENT**

A uniform beam PQ of length 100cm and of weight 35N lies on a support 40cm from the end P. Weights 54N and W are attached to the ends P and Q respectively to keep the beam in equilibrium. Find the value of W, to the nearest whole number.

**READING ASIGNMENT**:

Read Rotational Equilibrium and Principle of Moments. Page 178-185 of Further Mathematics Project III.

**WEEK EIGHT**

**FRICTION**

Basic Concept

Coefficient of Friction

Forces Acting on a Body

**Basic Concept**: When two bodies are in contact, each one is exerting a force on the other. Therefore, friction can be defined as the force which tends to oppose the relative sliding motion of two surfaces in contact. Frictional force is the opposing force between two forces in contact.

The direction of friction is opposite to the direction in which the motion will occur. The frictional force for any two surfaces in contact has a value given by”

F = UR

Where R is the normal reaction between the bodies and obtained as; R = m x g

U = Coefficient of friction and the value depends only on the nature of the surfaces in contact.

**Examples**

1: A mass of 4kg rests on a rough horizontal table, with U = 0.4. Find the least force sufficient to move the mass: (take g = 10ms-2)

**Solution**

Sufficient force; F = UR.

R = m x g = 4 x 10 = 40

F = 0.4 x 40

F = 16N

2. If a force of 10N is just sufficient to move a mass of 2kg resting on a rough horizontal table, find the coefficient of friction (g = 10ms-2)

**Solution**

F = 10 N , m = 2kg.

F = UR

10 = U x 2 x 10

10 = 20U

u = 10/20

U = 0.5

**EVALUATION**

A body of mass 8kg rests on a horizontal surface. If the coefficient of friction between the body and the horizontal surface is 0.65, calculate the minimum horizontal force enough to just move the body.

**FORCES ACTING ON A BODY PLACED ON A ROUGH INCLINED PLANE**

Rough Inclined Plane: There are two basic forces acting on an object placed on an inclined plane; the applied forces P and the frictional force F P= Mg Sin Ө

F = Mg SinӨ

R = Mg Cos Ө

Recal F = *U*R

:. F = *U*MgCos Ө where F is the limiting

Friction:

The least force required in P to make the body move up the inclined plane r is P = UR + Mg Sin Ө

:. P = F + MgSinӨ

P = UR + MgSinӨ

The least force required to make the body slide down the plane is thus

P + f = mgSin Ө

P = mg Sin Ө- *U*r.

Where m = mass and Ө= angle of friction.

The value of the coefficient of fricton from F = *U*R and

MgSinӨ = *U*MgCos Ө is defined by

*U* = mg sin Ө

Mg cos Ө

:. U = tan Ө

**Examples**

1. A block is placed on an inclined plane at an angle of 30o to the horizontal and just remains at rest. Find the coefficient of friction.

Solution

Ө = 30o, U = tan Ө

U = tan 30o

U = 1/√3 or 0.5774.

1. A body of mass 4.5kg rests on a smooth plane inclined at an angle of 47o to the horizontal. Calculate the magnitude of the force P parallel to the plane just enough to prevent the body from sliding down the plane

(g= 9.8ms-2).

Solution

P = mg Sin Ө

P = 4.5 x 9.8 x sin 47o = 44.1 x 0.7314.

P = 32.25N.

**EVALUATION**

A particle of mass 25kg slides down a rough plane inclined at angle 30o to the horizontal. If the coefficient of friction is 0.2, find, in ms-2 the acceleration of the particle correct to 3 significant figures (take g = 10ms-2)

**GENERAL EVALUATION**

A uniform ladder of length 6m and mass 30kg rests with one end against a rough vertical wall and the other end on a rough horizontal ground. The coefficient of friction at each point of contact is 0.3. If the ladder is on the point of slipping, calculate the (i) normal reaction of the ground (ii) frictional force at the wall (iii) angle of inclination of the ladder to the ground. (Take g = 10ms-2)

**READING ASIGNMENT**

Read Friction page 191 -195

**WEEKEND ASSIGNMENT**

1. A body is in limiting equilibrium on a plane inclined at an angle to the horizontal. if Cos = 0.8, calculate the coefficient of friction.

2. A big stone is of mass 13kg. A boy whose weight is 59N sits on the stone. Find the minimum horizontal force P required to move the stone on the ground. If the coefficient of friction is 0.25 (g = 9.8ms-2).

3. A mass of 8kg tests on a rough table with U = 0.6. Find the least force which will make the mass move (g = 10ms-2).

4. A mass of 10kg slides down a rough plane inclined at Ө to the horizontal where sin Ө = 0.6. if *U* = 0.3, find the acceleration of the box.

5. A body can just rest in equilibrium on a slope inclined at Ө to the horizontal where sinӨ = 5/13, find *U*